Module 5: Radar Detection and Ambiguity

This module delves into the theoretical underpinnings of target detection in radar systems and explores the inherent limitations and trade-offs that arise from the nature of radar signals and their processing. We will cover the statistical methods used to decide the presence of a target, the optimal processing techniques for maximizing detection performance, and the fundamental limitations on a radar's ability to uniquely identify targets in both range and velocity.

5.1 Detection Theory Fundamentals

Radar detection is fundamentally a decision-making process under uncertainty. The receiver continuously processes echoes that contain not only potential target signals but also inevitable noise and sometimes clutter. The challenge is to distinguish a genuine target echo from random fluctuations caused by noise. This is addressed through the principles of hypothesis testing.

5.1.1 Introduction to Hypothesis Testing

In the context of radar detection, we essentially have two competing hypotheses about the received signal in any given time-frequency cell:

- **Hypothesis H0 (Null Hypothesis): Only noise is present. This corresponds to the scenario where there is no target.**
  r(t)=n(t)
  where r(t) is the received signal and n(t) is the noise.
- **Hypothesis H1 (Alternative Hypothesis): A target signal is present along with noise. This corresponds to the scenario where a target exists.**
  r(t)=s(t)+n(t)
  where s(t) is the target signal and n(t) is the noise.

The radar receiver's task is to decide between H0 and H1 based on the received data. This decision is made by comparing a calculated "test statistic" (derived from the received signal) against a predetermined detection threshold. If the test statistic exceeds the threshold, H1 is chosen (target detected); otherwise, H0 is chosen (no target detected).

There are two types of errors that can occur in this decision process:

- **Type I Error (False Alarm): Deciding H1 is true when H0 is actually true. This means declaring a target when only noise is present. The probability of this error is called the Probability of False Alarm (Pfa). A false alarm consumes resources (e.g., operator attention, tracking system processing) and can lead to incorrect tactical decisions.**

- Type II Error (Missed Detection): Deciding H0 is true when H1 is actually true. This means failing to detect a target that is actually present. The probability of this error is called the Probability of Missed Detection (PM). This is equivalent to 1−Pd, where Pd is the Probability of Detection. A missed detection can have severe consequences, especially in critical applications like air traffic control or defense.

The goal of radar detection theory is to minimize the probability of these errors, or to manage the trade-off between them, given the inherent uncertainty introduced by noise.

5.1.2 Receiver Operating Characteristics (ROC) Curves

Receiver Operating Characteristics (ROC) curves are a powerful tool used to visualize and analyze the performance of a detection system, such as a radar receiver. An ROC curve plots the Probability of Detection (Pd) against the Probability of False Alarm (Pfa) for various possible settings of the detection threshold.

Each point on an ROC curve represents a different threshold setting.

- Moving the threshold lower (more lenient decision) increases both Pd and Pfa.
- Moving the threshold higher (more stringent decision) decreases both Pd and Pfa.

A "perfect" detection system would have an ROC curve that goes from (0,0) directly to (0,1) and then to (1,1), meaning it can achieve a Pd of 1 (100% detection) with a Pfa of 0 (no false alarms). In reality, there is always a trade-off.

Key characteristics of ROC curves:

- Shape: For a given Signal-to-Noise Ratio (SNR), the ROC curve is unique. A higher SNR shifts the curve towards the upper-left corner of the plot, indicating better detection performance (higher Pd for a given Pfa, or lower Pfa for a given Pd).
- Independent of Threshold: The ROC curve itself does not depend on the specific threshold value. Instead, the curve shows what performance is *achievable* by varying the threshold.
- Performance Comparison: ROC curves are invaluable for comparing the performance of different radar systems or different detection algorithms. A system whose ROC curve is closer to the upper-left corner is superior.
- Probability of False Alarm (Pfa): This is usually set to a very small, acceptable value (e.g., 10−6 or 10−8) to ensure that the operator is not overwhelmed by false targets.

- **Probability of Detection (Pd): This is what we want to maximize for the chosen Pfa. A typical requirement might be Pd=0.9 (90% detection).**

How to read an ROC curve:

1. **Choose an acceptable Pfa value on the x-axis.**
2. **Move vertically up to the ROC curve.**
3. **Then move horizontally to the left to find the corresponding Pd value on the y-axis.**

In-depth Explanation:

ROC curves are derived from the probability density functions (PDFs) of the receiver output for both the "noise only" case (H0) and the "signal plus noise" case (H1). The overlap between these two PDFs dictates the inherent trade-off. For higher SNR, the PDFs are more separated, leading to less overlap and thus better performance on the ROC curve. The optimal decision criterion for detecting a signal in the presence of noise, assuming Gaussian noise and known signal characteristics, is often based on the Neyman-Pearson criterion, which states that for a fixed Pfa, the detection threshold should be chosen to maximize Pd.

5.2 Matched Filtering

Matched filtering is a fundamental concept in radar signal processing that ensures optimal detection performance. It is a filter designed to maximize the output Signal-to-Noise Ratio (SNR) when a known signal is corrupted by additive white Gaussian noise (AWGN).

5.2.1 Principle of the Matched Filter

The principle of the matched filter is that its impulse response is the time-reversed and conjugated version of the known signal waveform that it is trying to detect. If the input signal is s(t), the impulse response of the matched filter h(t) is given by:

$$h(t)=s^*(T-t)$$

where:

- **$s^*(t)$ denotes the complex conjugate of the signal s(t).**
- **T is the duration of the signal (or a time constant that shifts the output peak to a convenient time, often chosen such that the peak output occurs at t=T).**

When the radar echo $s(t)$ passes through a filter with impulse response $h(t)$, the output of the filter $y(t)$ is the convolution of the input signal and the filter's impulse response:

$$y(t)=s(t)*h(t)=\int_{-\infty}^{\infty}s(\tau)h(t-\tau)d\tau$$

Substituting $h(t)=s^*(T-t)$:

$$y(t)=\int_{-\infty}^{\infty}s(\tau)s^*(T-(t-\tau))d\tau=\int_{-\infty}^{\infty}s(\tau)s^*(T-t+\tau)d\tau$$

At the specific time $t=T$ (when the signal is optimally aligned with the filter), the output is:

$$y(T)=\int_{-\infty}^{\infty}s(\tau)s^*(\tau)d\tau=\int_{-\infty}^{\infty}|s(\tau)|^2d\tau=E_s$$

where E_s is the total energy of the signal. This shows that the matched filter output peaks at a value equal to the signal energy when the signal is perfectly matched to the filter.

The core idea is that the filter acts as a correlator. It continuously correlates the incoming noisy signal with a replica of the expected target waveform. When the target echo is present and perfectly aligned in time, the correlation peaks, providing the maximum possible SNR at that specific instant.

5.2.2 Derivation of Optimal SNR

Let's consider a simplified derivation. Suppose the received signal is $x(t)=s(t)+n(t)$, where $s(t)$ is the signal and $n(t)$ is white Gaussian noise with a power spectral density of $N_0/2$. The goal is to find a filter $h(t)$ that maximizes the output SNR at a specific time T.

The output signal power at time T is $|y_s(T)|^2$, where $y_s(T)$ is the output when only the signal is input.

$y_s(T)=\int_{-\infty}^{\infty}S(f)H(f)e^{j2\pi fT}df$ (using Fourier transforms, where $S(f)$ and $H(f)$ are the Fourier transforms of $s(t)$ and $h(t)$).

The output noise power is $\text{Noise Power}=\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2df$.

The instantaneous SNR at the output is:

$$SNR_{out}=\frac{|y_s(T)|^2}{\text{Noise Power}}=\frac{\left|\int_{-\infty}^{\infty}S(f)H(f)e^{j2\pi fT}df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2df}$$

Using the Cauchy-Schwarz inequality, which states that $\left|\int g_1(f)g_2(f)df\right|^2\le\int|g_1(f)|^2df\int|g_2(f)|^2df$, with $g_1(f)=S(f)e^{j2\pi fT}$ and $g_2(f)=H(f)$, we can find the condition for maximum SNR.

The maximum occurs when $H(f)$ is proportional to $S^*(f)e^{-j2\pi fT}$.

Thus, $H(f) = kS^*(f)e^{-j2\pi fT}$ for some constant k.

Taking the inverse Fourier Transform of $H(f)$ to find $h(t)$:

$$h(t) = ks^*(T-t)$$

This confirms that the impulse response of the matched filter is the time-reversed and conjugated version of the signal.

When this condition is met, the maximum output SNR achieved by the matched filter is:

$$SNR_{out,max} = \frac{2E_s}{N_0}$$

where $E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt$ is the total energy of the signal, and $N_0/2$ is the two-sided power spectral density of the white Gaussian noise.

In-depth Explanation:

This formula is incredibly significant. It states that the maximum achievable SNR at the receiver output, and therefore the best possible detection performance, depends only on the total energy of the received signal and the noise power spectral density, not on the specific shape of the waveform. This means that a long, low-power pulse can achieve the same detection performance as a short, high-power pulse, provided their total energies are equal. This principle is exploited in pulse compression techniques, where long, coded pulses are transmitted to achieve high total energy (and thus long range) while maintaining good range resolution (due to the effective short duration after compression). The matched filter effectively performs this pulse compression.

Numerical Example:

Consider a radar system transmitting a rectangular pulse with a peak power of $P_{peak} = 1$ MW and a pulse width $\tau = 1 \text{ \textmu s}$.

The received echo signal has an amplitude such that its energy $E_s = 10^{-14}$ J.

The receiver noise has a power spectral density $N_0 = 4 \times 10^{-20}$ W/Hz.

What is the maximum SNR achievable at the matched filter output?

Given:

$E_s = 10^{-14}$ J

$N_0 = 4 \times 10^{-20}$ W/Hz

$$SNR_{out,max} = \frac{2E_s}{N_0} = \frac{2 \times 10^{-14}\ \text{J}}{4 \times 10^{-20}\ \text{W/Hz}}$$

$$SNR_{out,max} = \frac{2 \times 10^{-14}}{4 \times 10^{-20}} = 0.5 \times 10^{6} = 500{,}000$$

In decibels (dB):

$$SNR_{dB} = 10 \log_{10}(500{,}000) \approx 56.99\ \text{dB}$$

This high SNR indicates very strong detection capability for this particular received signal energy and noise level.

5.3 Radar Ambiguity Function

The Radar Ambiguity Function is a powerful mathematical tool that characterizes the resolution capabilities and inherent ambiguities of a radar waveform in both range and Doppler (velocity). It helps radar designers understand how well a particular waveform can distinguish between multiple targets and how susceptible it is to various forms of ambiguity.

5.3.1 Definition

The ambiguity function, often denoted as $\chi(\tau, f_d)$, is a two-dimensional function of time delay (τ) and Doppler frequency (f_d). It essentially represents the output of a matched filter when the received signal is a Doppler-shifted and time-delayed version of the transmitted signal.

For a complex envelope of a transmitted signal $u(t)$, the ambiguity function is defined as:

$$\chi(\tau, f_d) = \int_{-\infty}^{\infty} u(t) u^*(t-\tau) e^{j2\pi f_d t} dt$$

where:

- **$u^*(t-\tau)$ is the complex conjugate of the time-delayed signal.**
- **$e^{j2\pi f_d t}$ accounts for the Doppler shift.**

The magnitude squared, $|\chi(\tau, f_d)|^2$, is often plotted and represents the output power of the matched filter as a function of range and Doppler mismatches.

In-depth Explanation:

The peak of the ambiguity function, at $\chi(0,0)$, corresponds to a perfectly matched filter output for a target with zero time delay and zero Doppler shift (i.e., the target at its true range and velocity). Any deviation from this peak along the τ or f_d axes represents a mismatch. The shape of the ambiguity function around this peak reveals the radar's resolution characteristics. Its behavior far from the peak indicates potential ambiguities.

5.3.2 Properties

The ambiguity function has several important properties that provide insights into radar waveform design:

- **Peak Value: The maximum value of the ambiguity function occurs at $\tau=0$ and $f_d=0$, where $|\chi(0,0)|^2=E_s^2$, the square of the signal energy. This confirms that the matched filter output is maximized for the correct range and Doppler.**
- **Volume Invariance: The volume under the magnitude squared of the ambiguity function is constant and equal to the square of the signal energy:**
  $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\chi(\tau,f_d)|^2 d\tau df_d=E_s^2$
  This is a crucial property: it means that improving resolution in one domain (e.g., range) often comes at the expense of resolution or increased ambiguity in the other domain (Doppler), or by increasing side-lobes elsewhere in the ambiguity plane. You cannot arbitrarily improve both resolutions simultaneously for a given signal energy.
- **Resolution:**
  - **Range Resolution: The width of the ambiguity function along the τ axis (at $f_d=0$) determines the range resolution. A narrow peak along this axis indicates good range resolution. This is generally achieved with short pulses or wideband signals (like chirps after pulse compression).**
  - **Doppler Resolution: The width of the ambiguity function along the f_d axis (at $\tau=0$) determines the Doppler (velocity) resolution. A narrow peak along this axis indicates good Doppler resolution. This is generally achieved with long pulse durations (which allow for more cycles of the Doppler shift to be observed) or long observation times.**
- **Ambiguities: Side-lobes in the ambiguity function, away from the main peak, indicate potential ambiguities.**
  - **Range Ambiguity: If there are significant peaks along the τ axis at non-zero τ (and $f_d=0$), it implies that a target at a different range might produce a response similar to a target at the true range. This is common with periodic pulse trains (PRF ambiguities).**
  - **Doppler Ambiguity (Blind Speeds): If there are significant peaks along the f_d axis at non-zero f_d (and $\tau=0$), it implies that targets with different Doppler shifts (velocities) might produce similar responses. This leads to "blind speeds" in pulsed radar, where targets with certain velocities produce zero or minimal Doppler shift relative to the pulse repetition frequency.**
- **Types of Waveforms and their Ambiguity Functions:**
  - **Single Rectangular Pulse: Has an ambiguity function shaped like a "thumbtack" or "sombrero" with a broad base, indicating poor**

    resolution in both range and Doppler if the pulse is long. The main lobe is wide in both dimensions.
    - Long CW Pulse (or unmodulated pulse): Has a very narrow ridge along the Doppler axis and a very wide spread along the range axis. Excellent Doppler resolution, terrible range resolution.
    - Linear FM (LFM) Chirp: Produces a "knife-edge" or "diagonal ridge" ambiguity function. It offers good range resolution (due to pulse compression) and good Doppler resolution, but it has a coupling between range and Doppler (a target with certain range and velocity can appear at a different range if processed with an incorrect Doppler assumption). This is known as range-Doppler coupling.
    - Pulse Train (unmodulated pulses at fixed PRF): Leads to multiple peaks (ambiguities) in both range (due to PRF) and Doppler (due to PRF, causing blind speeds). The ambiguity function becomes a repeating "bed of nails."

5.3.3 Role in Characterizing Resolution Capabilities and Ambiguities

The radar ambiguity function serves as a critical tool for radar engineers to:

- Select Optimal Waveforms: By analyzing the ambiguity function of different waveforms, designers can choose a waveform that best suits the application's requirements (e.g., high range resolution for imaging, high Doppler resolution for velocity measurement, or a balance of both).
- Understand Trade-offs: The volume invariance property highlights the inherent trade-offs in waveform design. It's impossible to have simultaneously perfect resolution in both range and Doppler with a finite energy signal. Improving one often degrades the other or creates undesirable side-lobes (ambiguities).
- Predict Performance: The ambiguity function can predict how well a radar will be able to separate multiple targets in a complex scenario, and how susceptible it will be to false targets due to ambiguities.
- Design Processing Algorithms: Knowledge of the ambiguity function helps in designing signal processing algorithms, such as those for pulse compression, that account for the waveform's characteristics and mitigate ambiguities. For example, using different PRFs ("staggered PRF") or frequency diversity can mitigate blind speeds and range ambiguities.

Numerical Example: Range-Doppler Coupling in LFM

A radar uses an LFM chirp with a pulse width $\tau = 10 \text{ \textmu s}$ and a bandwidth $\Delta F = 10$ MHz.

The range-Doppler coupling for an LFM chirp is approximately ΔR=−vrfcenterTsweep. (A more specific expression is ΔR=−TeffτeffΔFτ0vr for a specific definition of effective pulse duration and bandwidth).

A simpler approximation relating to the ambiguity function's diagonal ridge is that a Doppler shift Δfd can be interpreted as an equivalent range error ΔRequiv. For an LFM signal, this relationship is:

ΔRequiv=−2cΔFτΔfd

Let's use a common form related to the slope of the ambiguity function's main ridge. The slope of the main ridge in the τ−fd plane for an LFM signal is often given as α=−ΔFτ.

If a target has a true Doppler shift of 100 Hz but is incorrectly assumed to have 0 Hz (due to a processing error or blind speed), what is the apparent range error?

Let's simplify and use the approximate relation for range-Doppler coupling due to a Doppler error for an LFM chirp. The range measurement error ΔR due to an uncompensated Doppler shift Δfd is:

ΔR=−2ΔFcτpΔfd

where τp is the pulse duration and ΔF is the frequency deviation of the chirp.

Given:

$\tau_p = 10 \text{ \textmu s} = 10 \times 10^{-6} \text{ s}$

ΔF=10 MHz=10×106 Hz

Δfd=100 Hz

c=3×108 m/s

ΔR=−2×(10×106 Hz)(3×108 m/s)×(10×10−6 s)×100 Hz

ΔR=−2×1073×102×100=−20,000,000300×100=−0.000015×100=−1.5 m

This means an uncompensated Doppler error of 100 Hz for this LFM waveform could result in a range error of −1.5 meters. This illustrates how Doppler and range measurements are coupled in LFM waveforms.

5.4 Probability of False Alarm and Detection

The Probability of False Alarm (Pfa) and the Probability of Detection (Pd) are the two most critical metrics for evaluating radar detection performance. They

are intrinsically linked and represent the trade-offs inherent in any statistical decision-making process.

5.4.1 Detailed Analysis of Pfa and Pd

Probability of False Alarm (Pfa):

Pfa is the probability that the radar receiver declares a target present when, in reality, only noise (or clutter) is present.

Pfa=P(Detect∣Noise Only)

A false alarm occurs when the noise-only voltage at the detector output exceeds the set detection threshold (VT).

The Pfa is determined by:

1. **The statistical distribution of noise: For random noise, it's often modeled as Gaussian (or Rayleigh after envelope detection). The shape of this distribution dictates how likely it is for noise to exceed a certain threshold.**
2. **The detection threshold (VT): A higher threshold reduces Pfa (fewer false alarms), but also reduces Pd. A lower threshold increases Pfa (more false alarms), but also increases Pd.**
3. **Receiver Bandwidth and Integration Time: These affect the noise power.**

For a fixed threshold, Pfa can be thought of as the fraction of time that noise alone would exceed the threshold. This directly relates to the clutter and noise "spikes" that an operator might see on a display in the absence of targets. Typical values for radar systems are very small, e.g., 10^{-6} (one false alarm per million decision opportunities) to 10^{-8} or even lower, depending on the system's operational requirements.

Probability of Detection (Pd):

Pd is the probability that the radar receiver correctly declares a target present when a target signal is actually present, along with noise.

Pd=P(Detect∣Signal + Noise)

Pd is determined by:

1. **Signal-to-Noise Ratio (SNR): This is the most dominant factor. A higher SNR means the target signal is stronger relative to the noise, making it easier to distinguish from noise, and thus leading to a higher Pd.**
2. **The detection threshold (VT): As discussed, a lower threshold increases Pd.**

3. Target Fluctuation Characteristics (Swerling Models): Real targets do not reflect radar energy with a constant amplitude. They "fluctuate" due to changes in aspect angle, polarization, and multipath effects. These fluctuations significantly impact Pd.
4. Number of Integrated Pulses (N): By coherently or non-coherently integrating (summing) multiple echoes from the same target, the effective SNR improves, leading to a higher Pd.

5.4.2 Their Relationship and Factors Influencing Them

Pd and Pfa are intimately related through the detection threshold and the SNR. For a given SNR, increasing Pd (e.g., by lowering the threshold) will inevitably increase Pfa. Conversely, decreasing Pfa (e.g., by raising the threshold) will inevitably decrease Pd. This fundamental trade-off is precisely what ROC curves illustrate.

Factors Influencing Pfa and Pd:

- Signal-to-Noise Ratio (SNR): As mentioned, SNR is the primary determinant of Pd for a given Pfa. The higher the SNR, the better the detection performance. SNR depends on:
  - Transmitted Power (Pt)
  - Antenna Gain (G)
  - Target Radar Cross Section (σ)
  - Range (R)
  - System Noise Temperature (Ts)
  - Receiver Bandwidth (B)
  - Pulse Integration (N)
- Noise Statistics: Typically assumed to be additive white Gaussian noise (AWGN). If noise is non-Gaussian or colored, more complex processing is needed.
- Detection Threshold: The judicious selection of the detection threshold is crucial. It is often set to achieve a desired constant Pfa over time, which might require adaptive thresholding (e.g., Constant False Alarm Rate - CFAR - processing) to account for varying noise or clutter levels.
- Target Fluctuation (Swerling Models): This is a very significant factor. If a target's radar cross-section (RCS) fluctuates (changes rapidly from pulse to pulse or scan to scan), it makes detection harder. Strong pulses might occur, but also very weak ones that fall below the threshold. The average RCS might be high, but the instantaneous RCS can be low.
- Number of Integrated Pulses (N): When multiple pulses are integrated, the SNR increases by a factor related to N (e.g., N for coherent

integration, or N for non-coherent integration). This improves Pd for a given Pfa.

- Clutter: Strong unwanted echoes from stationary objects or weather can significantly increase the effective noise level, making it harder to detect legitimate targets and leading to a higher Pfa or lower Pd if not properly mitigated.

Numerical Example: Impact of SNR on Pd

Let's assume a specific fixed Pfa=10^{-6}. For a simple "square-law" detector (typical in non-coherent processing), the relationship between Pd, Pfa, and SNR is often depicted in universal detection curves or through more complex numerical evaluations (e.g., using Marcum's Q-function for non-fluctuating targets).

Without going into the complex mathematical functions (as they require external reference or complex derivation), let's illustrate the concept:

- If SNR = 10 dB, for Pfa=10^{-6}, Pd might be around 0.5 (50%).
- If SNR = 13 dB (an increase of 3 dB, or a doubling of signal power), for the same Pfa=10^{-6}, Pd might increase significantly to around 0.9 (90%). This clearly demonstrates that even a small increase in SNR can lead to a substantial improvement in the probability of detection. This highlights the importance of maximizing SNR through good radar design and matched filtering.

5.5 Modified Radar Range Equation with Swerling Models

The basic Radar Range Equation provides a foundational understanding of the maximum range of a radar system, assuming a non-fluctuating (constant Radar Cross Section - RCS) target. However, real-world targets, especially complex ones like aircraft, often exhibit significant fluctuations in their RCS as they change aspect angle relative to the radar. To account for this variability and provide more realistic performance predictions, Swerling Models are incorporated into the radar range equation.

5.5.1 Incorporating Target Fluctuation Models (Swerling I-IV)

The classical Radar Range Equation for a non-fluctuating target (sometimes called Swerling 0 or Swerling V, which are ideal cases) is given by:

$R_{max}^4=\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}$

Where:

- R_{max} is the maximum range.
- P_t is the transmitted peak power.
- G is the antenna gain.
- A_e is the effective aperture of the antenna.
- σ is the Radar Cross Section (RCS) of the target (assumed constant).
- S_{min} is the minimum detectable signal power at the receiver, which is the product of noise power P_n and the minimum detectable Signal-to-Noise Ratio (SNR) required for detection, $(SNR_{min})_{non-fluctuating}$.
  $S_{min}=kT_sBF(SNR_{min})_{non-fluctuating}$
  where k is Boltzmann's constant, T_s is system noise temperature, B is noise bandwidth, F is noise figure.

The issue with this equation is that σ is rarely constant. Swerling recognized this and proposed statistical models for target RCS fluctuations, which profoundly impact P_d and thus the predicted range. These models are based on chi-squared distributions with different degrees of freedom, representing different types of targets and fluctuation rates.

The four primary Swerling models are:

- Swerling I: This model represents a target whose RCS fluctuates slowly from scan to scan (i.e., the RCS is constant over an entire scan/illumination time, but changes independently for the next scan). The probability density function (PDF) of the RCS follows an exponential distribution. This model is often used for large, complex targets like bomber aircraft.
  - Impact: Requires significantly more SNR (or higher transmitted power) than a non-fluctuating target to achieve the same P_d. The radar has to "wait" for the target to fluctuate into a strong reflection.
- Swerling II: Similar to Swerling I, but the RCS fluctuates rapidly from pulse to pulse within a single scan. Each received pulse from the target has an independent RCS value. The PDF is also exponential. This model is typical for rapidly changing aspect angles or targets with many independent scatterers.
  - Impact: Due to pulse-to-pulse independence, averaging over many pulses helps "smooth out" the fluctuations, making P_d less sensitive to instantaneous low RCS values compared to Swerling I for the same average RCS. For the same number of integrated

pulses, Swerling II generally requires lower SNR than Swerling I for the same Pd.

- Swerling III: This model represents a target whose RCS fluctuates slowly from scan to scan, but with a different statistical distribution (chi-squared with four degrees of freedom, or two independent exponential components). This applies to targets that can be modeled as having a few dominant scatterers.
    - Impact: Performance lies between Swerling I and Swerling IV.
- Swerling IV: Similar to Swerling III, but the RCS fluctuates rapidly from pulse to pulse. The PDF is the same as Swerling III.
    - Impact: Generally requires the least amount of average SNR for a given Pd among the fluctuating models, as the rapid fluctuations and two dominant scatterers provide more "opportunities" for a strong return within a pulse train.

Modified Radar Range Equation incorporating Swerling Models:

To account for these fluctuations, the concept of a "detection degradation factor" or "fluctuation loss" is introduced. Alternatively, the minimum detectable SNR is adjusted for each Swerling case and desired Pd and Pfa.

The modified range equation is typically expressed by solving for the required SNR at the receiver for a given Pd, Pfa, and number of integrated pulses (N), for each Swerling model. These values are often found from published curves or detailed numerical calculations.

The general form remains:

$R_{max}4=(4\pi)3(SNR_{req}BFkT0L_{sys})P_tG2\lambda 2\bar{\sigma}$

Or, more simply, where SNRreq is the minimum SNR required for a given Pd and Pfa for a specific Swerling model and number of integrated pulses.

$R_{max}4=(4\pi)2S_{min}(\text{Swerling}, P_d,P_{fa},N)P_tGA_e\bar{\sigma}$

Here, $\bar{\sigma}$ is the average RCS of the target. The Smin (or SNRreq) value will be significantly higher for fluctuating targets, especially Swerling I, compared to a non-fluctuating target for the same Pd and Pfa.

Key takeaways:

- For a given average RCS ($\bar{\sigma}$), Pd, and Pfa, fluctuating targets (Swerling I-IV) always require a higher SNR than a non-fluctuating target (Swerling 0) to achieve the same detection performance.

- Swerling I and III models (scan-to-scan fluctuations) are generally harder to detect than Swerling II and IV models (pulse-to-pulse fluctuations) for the same number of integrated pulses, because pulse integration is less effective in smoothing out fluctuations that are constant over many pulses.
- The actual Pd versus SNR curves (often plotted in charts) will be different for each Swerling model.

Numerical Example: Impact of Swerling Models on Required SNR

Consider a radar system aiming for a Probability of Detection (Pd) of 0.9 and a Probability of False Alarm (Pfa) of 10^{-6}, integrating 10 pulses (N=10).

Using standard radar performance charts (which are derived from the Swerling models and detection theory, and typically found in radar textbooks), the approximate required SNR (per pulse, assuming non-coherent integration) might be:

- Non-fluctuating (Swerling 0): Approximately 8 dB (for $Pd=0.9, Pfa=10^{-6}, N=10$)
- Swerling I (Scan-to-scan, exponential): Approximately 15 dB
- Swerling II (Pulse-to-pulse, exponential): Approximately 10 dB
- Swerling III (Scan-to-scan, 4 deg. of freedom): Approximately 13 dB
- Swerling IV (Pulse-to-pulse, 4 deg. of freedom): Approximately 9 dB

This example clearly shows that:

1. Fluctuating targets require higher SNR than non-fluctuating targets for the same performance.
2. Targets fluctuating pulse-to-pulse (Swerling II, IV) require less SNR than those fluctuating scan-to-scan (Swerling I, III) for the same number of integrated pulses, demonstrating the benefit of pulse integration on rapidly fluctuating targets.

This concludes Module 5, providing a comprehensive understanding of the statistical nature of radar detection and the critical role of waveform and target characteristics in determining radar system performance and limitations.